Beyond rigidity: obtain localisability with noisy ranging measurement

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Abstract: Location awareness is crucial for wireless networks, attracting many research efforts in recent years. One of the fundamental problems of localisation is to determine whether or not a network is uniquely localisable. To address this problem, existing approaches often adopt the rigidity theory, which assumes accurate inter-node distance measurements. Considering the measurement errors, we introduce the concept of *strong localisability* and propose an algorithm, called LAS, to identify and locate a kind of strongly localisable networks. Besides, we further conduct extensive simulations to show the performance of LAS design.

Keywords: flip ambiguity; localisation; strongly localisable network; wireless networks.

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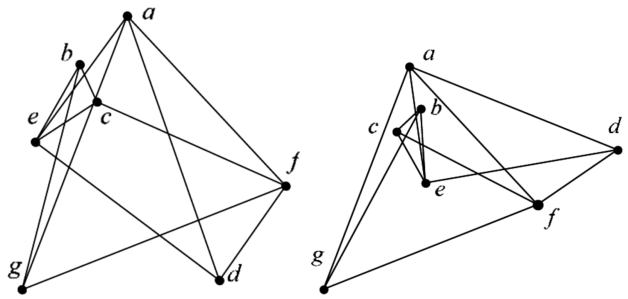
1 Introduction

Location awareness is crucial for wireless networks (Li and Liu, 2009), which provides essential contexts for data interpretation (Mo et al., 2009) and network operations (Li et al., 2008). Owing to the constraints of hardware and cost, location information is not an initially available knowledge. Instead, only a few nodes, called anchors, know their locations in the network set-up step. Then, other nodes compute their locations based on inter-node distance measurements. This procedure is called network localisation. For localisation, a fundamental problem is to determine the localisability of a network: whether the locations of the non-anchor nodes can be uniquely determined, given the distance-measurement and anchor information.

Recently, an increasing number of researchers study the localisability issue and achieve many results on identifying localisable networks (Aspnes et al., 2006; Eren et al., 2004). However, those results are based on rigidity theory, which relies on two essential assumptions. First, rigidity theory requires accurate ranging. Second, it demands the node distribution to be *generic*, which means any group of nodes do not lie in a proper subspace (e.g., any three nodes do not lie on a line). Unfortunately, neither of the assumptions can be fully satisfied in practice. Hence, rigidity theory can only indicate localisability in theory.

Figure 1(a) shows the ground truth of a network, where vertices denote the nodes and edges denote the associated nodes that can measure the mutual distances. If all distance measurements contain no error, this network is localisable according to rigidity theory. However, error is inevitable in practice for all ranging techniques and hardware platforms. Suppose the ranging error is not more than 10% of the real distance values. If error is considered, the originally localisable graph in Figure 1(a) has an alternative embedding that also satisfies all distance constraints shown in Figure 1(b). In this case, the existence of two feasible embeddings destroys the localisability of the example network. Researchers usually use the *stress* to evaluate the fidelity of embeddings (Goldenberg et al., 2006), defined as the squared discrepancy between the induced inter-node distances and the measured distances, i.e., $\sum_{(i,j)\in E}(\| p_i - p_j \| - \hat{d}_{ij})^2$, where E is the edge set, p_i is the realised position of vertex i and $\hat{d}_{ij}$ is the measured distance between node i and node j. In this example, the stress of Figure 1(a) is 4.1. In contrast, the stress of Figure 1(b) is 1.2 and will be chosen as the localisation result, leading to huge errors in location estimation. To summarise, purely structural rigidity fails to guarantee the network localisability under noisy measurement.

Figure 1 Two feasible realisations of a globally rigid graph with noisy ranging measurements



To highlight the impact of errors on localisability, we define a network is *weakly localisable* if the network is localisable under accurate ranging information. Rigidity theory can be used to solve this type of localisability. In contrast, a network is *strongly localisable* if the network has a unique realisation under noisy ranging measurement with bounded errors. In Section 2, we will give a formal definition of strongly localisable network, and show that a strongly localisable network has a unique realisation in case of errors.

According to the theory of strong localisability, we propose a Localisation Algorithm for Strongly localisable network, called LAS, to identify and locate a kind of strongly localisable networks: robust trilateration network. The insight of LAS is the equivalent conversion of the ranging errors. Compared with the existing robust localisation approaches, LAS has the following characteristics. First, LAS introduces very low cost on the localisation performance. Here, we define the *performance* of localisation algorithms as how many nodes can be successfully located in a given network. In other words, a high-performance algorithm can work well in sparse networks. The experiment results show that LAS can achieve a full localisation when the average degree is about 12. This result is of the same level with the traditional trilateration-based methods (Moore et al., 2004), which means LAS brings little performance degradation. Second, LAS is distributed and introduces low cost. The communicational and computational cost of LAS is both $O(n)$ in a network with bounded node degree,

where n is the number of nodes in the network. From the perspective of a single node, LAS only introduces $O(1)$ communicational and computational cost.

Major contributions of this work are as follows.

- We propose the concept of strong localisability, which accepts errors when studying localisability.
- We propose two versions of localisation algorithm (LAS and CLAS) to identify and locate robust trilateration networks, a special kind of strongly localisable networks. They both introduce low computational and communicational cost, thus are applicable in a resource-constrained network.
- We conduct extensive simulations to evaluate our design. The results show that LAS and CLAS can achieve robustness with very low performance cost.

The rest of this paper is organised as follows. We formally define the strongly localisable network and robust trilateration network in Section 2. In Section 3, we describe the rationale of LAS. In Section 4, we describe the implementation of LAS and CLAS design. Experimental studies are given in Section 5. We review the related work in Section 6 and conclude this work in Section 7.

2 Preliminary

In this section, we present the network model and the formal definition of strongly localisable network. Moreover, we also discuss the concept of robust trilateration network.

2.1 Strongly localisable network

Given a network and corresponding distance measurements, we use a distance graph $G = (V, E)$ to present the measurements, in which the vertices in V denote the nodes in the network and each edge $(i, j) \in E$ denotes node i and node j can measure the mutual distance. The corresponding measurement value and maximum error of each edge are presented by two functions: $d(i, j) : E \to R$ and $\delta(i, j) : E \to R$, respectively. We suppose a small portion of nodes, called anchors, are at known locations. Without loss of generality, m anchors are labelled from 1 to m, together with $n - m$ ordinary nodes labelled from $m + 1$ to n, where n denotes the total number of nodes in the network. The ground truth position of each node is denoted by p_i, $1 \le i \le n$. Hereinafter, we use a distance graph G as well as the two functions d and δ, i.e., (G, d, δ), to model a given network.

A *feasible realisation* of a network is a mapping $P : V \to R^2$ that guarantees $\left| \| P(i) - P(j) \| - d(i, j) \right| \le \delta(i, j)$ for all $(i, j) \in E$, where $\|.\|$ denotes the Euclidean distance of two positions. We say a feasible realisation is *globally rigid*, i.e., weakly localisable, if the distance-preserved mapping is unique in a plane (Aspnes et al., 2006). Then, we utilise this concept to define the strong localisability of a network.

Definition 1: A network is strongly localisable if all of its feasible realisations are globally rigid.

Strong localisability is an extension of weak localisability. Weak localisability purely concerns the qualitative attribute of the network structure, i.e., stability. In contrast, strong localisability also emphasises the quantitative attribute of the network structure. Clearly, weak localisability is a special case of strong localisability, where the bound of error $\delta(i, j) = 0$ for all $(i, j) \in E$. The ground truth of a network must be one of the feasible realisations of the corresponding graph (G, d, δ). Hence, if a network is strongly localisable, the localisation result will not suffer from the structural deformation.

2.2 Robust trilateration network

Location computation of weakly localisable networks is NP-hard (Aspnes et al., 2006), thus location computation of strongly localisable network is also NP-hard, motivating the research on location-computable networks (Fang et al., 2009). Among them, the trilateration network is well studied. Measuring the distances to three reference nodes at known locations, a target node can uniquely determine its location through trilateration. A network is call trilateration network if it can be located by a series of trilaterations. To obtain the computability and strong localisability simultaneously, we propose the concept of robust trilateration network.

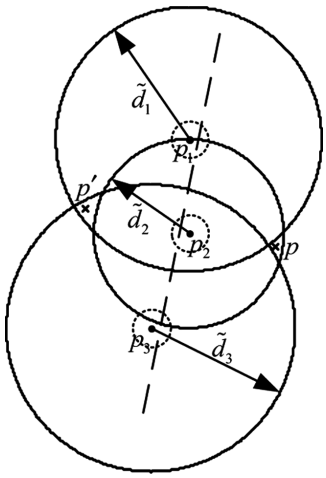
Definition 2: A robust trilaterative ordering for a graph G is an ordering of the vertices 1, 2, …, n, such that,

- the first m vertices are anchors
- from every vertex $i > m$, there are at least 3 edges to vertices earlier in the sequence, and all the induced subgraphs of the first i, $m < i \le n$, nodes are strongly localisable.

A graph with a robust trilaterative ordering is called a robust trilateration graph. And, its corresponding network is a robust trilateration network.

In the two-dimensional case, there are only two ambiguities preventing a rigid graph to be uniquely realised: flip ambiguity and flex ambiguity (Goldenberg et al., 2006). Fortunately, in trilateration graph, there is no chance for flex ambiguity, because redundantly rigidity is always satisfied in trilaterative extension. Hence, avoiding flip ambiguity is essential for achieving a robust trilateration network. Flip ambiguity is defined as that a target node has two possible positions corresponding to a ‘reflection’ across a set of mirror nodes (Goldenberg et al., 2006). Figure 2 shows an example of flip ambiguity in trilateration. When the reference nodes (p_1, p_2, and p_3) are approximately collinear, it is ambiguous to determine whether position p or p' (denoted by x-marks) is the correct location estimation of the target node.

Figure 2 Analysis of flip ambiguity



To obtain a robust trilateration network, we first generate a sub-network of all anchors, and then extend the sub-network by iteratively finding a node that fulfils the following conditions:

- the node has at least three distance measurements to nodes already in the sub-network
- the node can calculate its location without flip ambiguity.

3 Analysis of the flip ambiguity with noisy ranging measurement

In this section, we first discuss the rationale of the design of LAS, especially the idea of equivalent translation of the ranging errors. Then, we analyse the translated error and solve the problems along with it.

3.1 Equivalent translation of errors

We first show the equivalent translation of the measurement errors. Suppose we localise a target node by k ($k \geq 3$) distance measurements to the nodes at known locations, called reference nodes. Let a set of tuples $M = \{< p_i, \tilde{d}_i >\}$, $i = 1, 2, \ldots, k$, to denote the reference nodes, where p_i denotes the position of reference node i, and $\tilde{d}_i$ denotes the measured distance between the target node and the reference node i. Each $\tilde{d}_i$ contains two parts: the real distance between the node pair of d_i and the error of the measurement ε_i, i.e., $\tilde{d}_i = d_i + \varepsilon_i$. Suppose the exact position of the target node is given. We can translate the reference node ε_i distance along the direction of the distance measurement and we get the translated position $\tilde{p}_i$. After this step, we obtain a group of new measurement tuples $M' = \{< \tilde{p}_i, \tilde{d}_i >\}$, $i = 1, 2, \ldots, k$. The only difference between M and M' is that there are no measurement errors in M'. As a result, if all k translated positions $\tilde{p}_i$ in M' are not likely to be collinear, there is no chance of flip ambiguity.

In practice, neither the real position of the target node nor the measurement errors are available. Moreover, the positions of reference nodes may also contain errors. Hence, we cannot obtain the exact form of M'. Instead, we can estimate it by the distance measurement error and position estimation error. For a reference node i in M, suppose the maximum absolute value of the measurement error is $\delta_{d,i}$, and the maximum absolute value of position estimation error is $\delta_{p,i}$. Note that both of them may be a function of distances and other environmental parameters (Patwari et al., 2005). The translated position $\tilde{p}_i$ must lie in the *equivalent disk*, which is defined as a disk centred at p_i with a radius $r_i = \delta_{d,i} + \delta_{p,i}$. Then, we rewrite the tuples $< \tilde{p}_i, \tilde{d}_i >$ in M' to this form $< p_i, r_i, \tilde{d}_i >$. Let $\tilde{M}$ denote the set of the *equivalent measurement*, $\tilde{M} = \{< p_i, r_i, \tilde{d}_i >\}$, $i = 1, 2, \ldots, k$, which contains the possible positions of translated reference nodes and exact distance information.

Let p and p' denote the two possible positions due to a flip in M, then the perpendicular bisector of line segment pp' must pass through all $\tilde{p}_i$ in M'. Hence, the perpendicular bisector must intersect with all equivalent disks in $\tilde{M}$. As shown in Figure 2, we localise a target node by three measurements to the nodes indexed by 1, 2 and 3, respectively. If the two candidate positions denoted by x-mark are ambiguous by trilateration, then the perpendicular bisector of them (the dashed line in Figure 2) intersects with all the equivalent disks (the dashed circles in Figure 2). Hence, we obtain a property of flip ambiguity: when the trilateration suffers from flip ambiguity, the perpendicular bisector of the candidate positions must intersect with all equivalent disks of reference nodes.

This property provides an inspiration to avoid flips: if there does not exist any line that can intersect with all equivalent disks of reference nodes, then flip will not take place. For a given reference node set, we investigate the following Existence of Intersecting Line (EIL) problem to estimate the risk of flip ambiguity:

Given: A set of disks $S = \{<p_i, r_i>\}$, $i = 1, 2, \ldots, k$, in a plane.

Objective: To determine whether there exists a line intersecting with all disks in S.

3.2 EIL problem of equal radii

The EIL problem of equal radii focuses on a special case in which all the given disks are of the same size. Solving this issue can answer two questions in localisation:

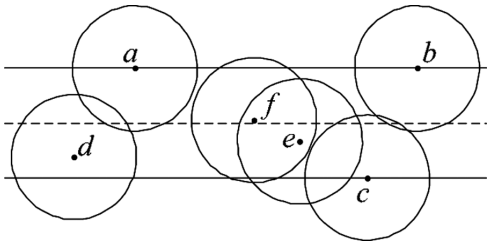
- When we indistinctively estimate the errors of the reference nodes, can the target node suffer from flip ambiguity?
- When there is no clear bound of the errors, how much error can the reference node set tolerate for avoiding flip ambiguity?

The following theorem shows the equivalent geometric property for solving the EIL problem with equal radii:

Theorem 1: *Given a set of disks* $S = \{<p_i, r_i>\}$*, if all the disks in S are of the same size, i.e.,* $r_i = r$*,* $i = 1, 2, \ldots, k$*, the EIL problem is equivalent to determine whether the width of the centres* $P = \{p_i\}$*, denoted as* $W(P)$*, is larger than* $2r$*. The width of P is defined as the minimum distance between parallel lines of support of P.*

Proof: Define the *medial line* of P as the line whose distances are at most $W(P)/2$ with the points in P. Take Figure 3 as an example, the distance of the solid lines shows the width of the point set, and the dashed line shows the medial line of the point set. If $W(P) \le 2r$, then the medial line will intersect all disks in S. Hence, there exists a line intersecting all disks. If there exists a line intersecting all disks in S, then the maximum distance between the line and the points in P is r. Hence, the width of P is less than or equal to $2r$, i.e., $W(P) \le 2r$. In a word, the answer of EIL problem of equal radii is equivalent to the condition $r > W(P)/2$. □

Figure 3 Width of a set and the medial line



We define the value $W(P)/2$ as the *error tolerance* of a reference node set. The error tolerance is computed by the width of the reference node set P. There are several high-efficient algorithms to compute the width of a point set, such as Rotating Caliper Algorithm (Houle and Toussaint, 1988). The algorithm mainly has three steps. First, we compute the convex hull of the point set. Clearly, the points determining the width of the set are definitely on the convex hull. Second, we find an arbitrary antipodal vertex–edge pair. We call an edge and a vertex of the convex hull an antipodal vertex–edge pair if parallel lines of support of P contain the edge and vertex. In Figure 3, vertex c and edge ab is an antipodal vertex–edge pair of the convex hull $abcda$. Third, we 'rotate' the initial antipodal vertex–edge pair to enumerate all the antipodal vertex–edge pairs of the convex hull, and report the half of the minimum distance of the antipodal vertex–edge pairs as the error tolerance, as shown in Algorithm 1.

Suppose there are k points in set P. Computing the convex hull of P in step 1 leads a complexity of $O(k\log k)$ time. Since there are at most k vertices or edges in CH, finding the initial antipodal vertex can be done in $O(\log k)$ time using binary search. Finally, the rotating calipers method can generate all the antipodal vertex–edge pairs in $O(k)$ time. Hence, the overall complexity of Algorithm 1 is $O(k\log k)$.

Algorithm 1 ErrorTolerance

Input: the positions of reference nodes P
Output: the maximum error which the reference node set can tolerate for flip ambiguity

1: Compute the convex hull of P
   $CH = ConvexHull(P)$
2: Find initial antipodal vertex-edge pair
   e = an arbitrary edge in CH
   $v = BinarySearch(e,CH)$
3: Use the rotating calipers to generate all antipodal vertex-edge pairs
   while existing edges in CH that have not been rotated **do**
      e = the clockwise neighboring edge of e in CH
      Rotate v clockwise until (v,e) form a antipodal vertex-edge pair
      $Width$ = the minimum distance of all the known (v,e) pair
4: **return** $width/2$

3.3 EIL problem of unequal radii

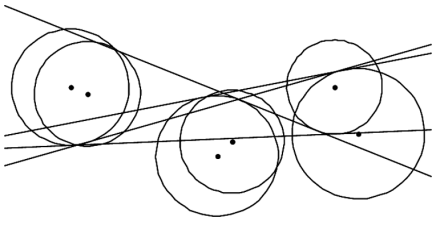
It is hard to solve the EIL problem of unequal radii by the macrostructure of the disks. Hence, we adopt boundary check to solve the EIL problem of unequal radii, as shown by Theorem 2.

Theorem 2: *Given a set of disks* $S = \{<p_i, r_i>\}$*,* $i = 1, 2, \ldots, k$*, the EIL problem is equivalent to determine that whether there exists a common tangent of two disks in S that intersects with all disks in S.*

Proof: Let $IL(S)$ denote the intersecting line set of the disk set S. The EIL problem is to determine whether $IL(S) = \varnothing$. Clearly, if $IL(S) = \varnothing$, there is no common tangent of two disks in S that intersects with all disks in S. Then, to analyse the case of $IL(S) \ne \varnothing$, we pre-process the disk set S. When a disk lies completely in another disk, any line intersecting with the inner disk will definitely intersect with the outer one. In this case, we can safely delete the outer disk from S. We use S' to denote the pre-processing result of S. Then, any pair of disks in S' have common tangents. In the following, we want to prove that there must exist a common tangent that intersects with all disks if $IL(S') \ne \varnothing$. The boundary of $IL(S')$ is a line that is about to violate the condition of intersecting with all disks in S'. Hence, a boundary line must be tangential to one disk. If the line is not tangential to any other disk, we rotate the line round the centre of the disk till it reaches the border of another disk. In this case, the line becomes a common tangent of two disks in S' and intersects with all disks. □

We show all boundary lines of a disk set in Figure 4. On the basis of Theorem 2, we propose an algorithm to solve the EIL problem with unequal radii, as shown in Algorithm 2.

Figure 4 The boundary of intersecting line set



Algorithm 2 EIL

Input: the set of disks $S=\{<p_i,r_i>\}$ $i=1,2,...,k$
Output: a Boolean value that indicates whether there exists a line intersecting with all disks in S.

1: Delete the outer disk of overlapped disks
   $S' = Preprocess(S)$
2: The trivial case
   if $|S'|\leq 2$ **then**
      return true
3: The extreme cases
   P' = the set of the centers of disks in S'
   $et = ErrorTolerance(P')$
   if $et \leq min(\{r_i\})$ **then**
      return true
   elseif $et>max(\{r_i\})$ **then**
      return false
4: Boundary checking
   for each disk pair $d_1,d_2\in S'$ $d_1\neq d_2$ **do**
      $CT = CommonTangent(d_1,d_2)$
      for each line $l\in CT$ **do**
         if l intersects with all disks in S' **then**
            return true
5: If it does not return in boundary checking step, then no intersecting line exists
   return false

Suppose there are k disks in set S. Step 1 needs to check all disk pairs and leads a complexity of $O(k^2)$ time. The size of S' is at most k. Step 3 calls the *ErrorTolerance* procedure in Algorithm 1 and leads a complexity of $O(k\log k)$ time. In fact, this step is optional and helps to make a direct decision in two extreme cases. The outer loop of Step 4 needs to repeat $O(k^2)$ times. The maximum number of common tangents of two disks is four, i.e., $|CT| \leq 4$ in Step 4. Thus, the inner loop of Step 4 repeats $O(1)$ times. Moreover, checking intersection of all disks costs $O(k)$ time. Hence, the boundary checking in Step 4 can be done in $O(k^3)$ time, and the overall complexity of Algorithm 2 is $O(k^3)$.

4 Implementation of LAS design

The design of EIL test is a robust complement of traditional localisation scheme. Hence, it can be directly applied to the traditional trilateration or the cluster-based localisation algorithms. We call these two implementations LAS and CLAS, respectively.

LAS inherits the merits of trilateration (Goldenberg et al., 2006; Eren et al., 2004): easy to implementation and fully distributed. The distributed trilateration is implemented as follows. First, nodes measure and record the distances with direct neighbours through the ranging devices. Second, anchor nodes broadcast their locations to their direct neighbours. Third, each node overhears the location broadcast of its neighbours. If the node can collect at least three location broadcasts of its neighbours, it computes the location of itself by trilateration. Then, it broadcasts its location to its direct neighbours. Finally, by waiting for sufficient long time, if a node still cannot collect three location broadcasts from its neighbours, it marks itself as an unlocalised node. LAS only need to enhance the third step to obtain robustness. In the third step, LAS requires the reference node set to pass the EIL test, before it performs a trilateration. Hence, LAS introduces very low additional cost when compared with traditional trilateration. If the number of neighbours in the network is bounded by a constant, the communicational cost and the computational cost are both $O(n)$, where n is the number of nodes in the network.

CLAS adopts the clusters to relax the requirements of anchor distribution. CLAS first generates a local coordinate system of a cluster. Then, nodes join to the cluster by trilateration, localising itself in the local coordinate system. Finally, CLAS localises the clusters having at least three anchors to the physical coordinate system by coordinate system registration (Horn et al., 1988). Clusters make the CLAS less depend on the anchor distribution, thus may increase the performance of localisation in sparse networks. Nevertheless, the way generating and converting coordinate system may introduce new sources of errors, thus it may potentially increase the error of the result. To obtain robustness, CLAS also requires nodes to pass the EIL test, when joining to the local clusters. Moreover, when CLAS localise the clusters, it demands the anchors to pass the EIL test to avoid global flip ambiguity. The detail of CLAS is shown in Algorithm 3.

Algorithm 3 CLAS

Input: the distance graph G
Output: the realized nodes of G

1: Generate local clusters
   while existing an unchecked triangle t in G **do**
2:    Check whether the triangle is robust
      if $EIL(t)$ **then continue**
3:    Generate a cluster
      $c = \{t\}$
      Generate the local coordinate system of c based on the triangle t
4:    Expend the cluster c
      while existing a node n in G can perform multilateration with reference node set ref in c **do**
         if not $EIL(ref)$ **then**
            $c = c\cup n$
5:    Add cluster to the cluster set C
      $C = C\cup c$
6: Realize the clusters
   while existing a cluster c in C contains anchors a, and $|a|\geq 3$ **do**
      if not $EIL(a)$ **then**
         Convert the local coordinate system of c to the physical coordinate system by coordinate system registration
7: For nodes which do not belong to clusters
   while existing an unlocated node n in G can perform multilateration with the realized node set ref **do**
      if not $EIL(ref)$ **then**
         Locate node n by multilateration
8: **return** the nodes in physical coordinate system

5 Experiment

In this section, we conduct extensive simulations to evaluate the proposed algorithms.

5.1 Experiment set-up

We generate uniformly random distributed network instances of 200 nodes in a square region. We randomly select a certain proportion of the deployed nodes as anchors. We use the parameter of distance measurement range to control the mean degree of the network. The distance information between neighbouring nodes is corrupted by additive white Gaussian noise (Liu et al., 2006) $N(0, \sigma^2)$ and we bounded the error by 3σ. We control the following parameters to compare the efficiency of LAS and CLAS:

- the mean degree of the network instances
- the proportion of anchors in the network
- the standard deviation of the distance-measurement noise.

We evaluate the efficiency of LAS and CLAS in three aspects. First, the number of successfully localised nodes indicates the performance of each algorithm. Second, the Standardised Position Estimation Error (SPEE) shows the error of the result. SPEE is defined as the percentage value between the mean position estimation error and the maximum distance-measurement range:

$$\text{SPEE} = \frac{1}{nR_{\max}} \sum_{i=1}^{n} \| p_i - \hat{p}_i \| \times 100\%,$$

where n is the total number of successfully localised nodes, $R_{\max}$ is the maximum distance measurement range, p_i is the real position of node i and $\hat{p}_i$ is the estimated position of node i. If $n = 0$, we define SPEE = 0. Third, the Standardised Distance Estimation Error (SDEE) indicates the fidelity between the estimated distance and the measured distance. SDEE is defined as the percentage value between the distance estimation error and the maximum distance-measurement range:

$$SDEE = \frac{1}{mR_{\max}} \sum_{i,j \in N, \tilde{d}_{ij} \in D} \left| \| \hat{p}_i - \hat{p}_j \| - \tilde{d}_{ij} \right|,$$

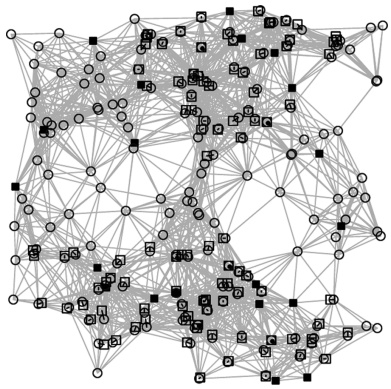
where N is the set of located nodes, $R_{\max}$ is the maximum distance measurement range, D is the set of distance measurement, $\hat{p}_i$ is the estimated position of node i and m is the total number of such measurement pairs. If $m = 0$, we define SDEE = 0.

We implement CLAS and LAS algorithm based on the error tolerance. We set the error tolerance threshold as the maximum possible error, i.e., 3σ. We compare the CLAS and LAS algorithms with the state-of-the-art design, Robust Quadrilaterals (RQ) (Moore et al., 2004). RQ shares the same design goal with us, which aims to acquire the structural stability with noisy ranging measurements. RQ acquires robustness by setting a threshold on the geometric property of the local four-node clusters. Stitching these robust local clusters, the resultant global cluster will not face structural deformation either. We implement full RQ algorithm with cluster optimisation in the cluster generation step, and the clusters are merged by the coordinate system registration (Horn et al., 1988). In the following sections, we will investigate the performance of LAS and CLAS, as well as the impact of cluster-based method on the accuracy of localisation.

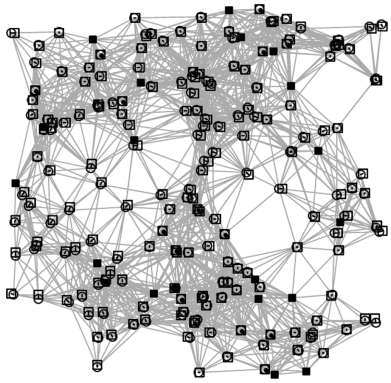
5.2 Overview of the algorithms

We demonstrate the execution of the RQ, LAS and CLAS algorithm in a randomly generated network instance with 200 nodes, and the average degree is about 20. We randomly select 10% nodes as anchors, and set the errors as much as 10% of the distance measurement value. Figure 5 shows the result of each algorithm, in which solid squares denote anchors, soft dots present the ground truth positions of the nodes, and the soft squares indicate the estimated positions of the nodes.

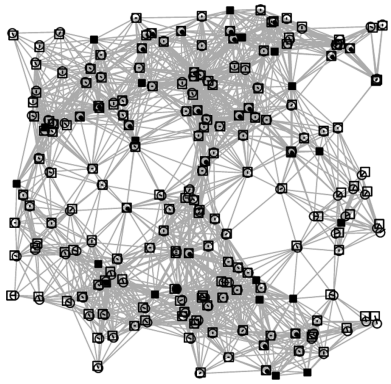
Figure 5 Overview of the algorithms: (a) RQ; (b) LAS and (c) CLAS



(a)



(b)



(c)

Figure 5(a) shows the result of RQ algorithm. It locates 103 out of the total 180 non-anchor nodes. The SPEE is about 4.7 and the SDEE is about 2.6. Figure 5(b) shows the result of LAS algorithm. It locates all the 180 non-anchor nodes. The SPEE is about 2.3 and the SDEE is about 1.3. Figure 5(c) shows the result of CLAS algorithm. It locates all the 180 non-anchor nodes. The SPEE is about 2.7 and the SDEE is about 1.4.

The RQ algorithm locates fewer nodes than the other two algorithms, because it requires generating uniquely distributed overlapped local clusters. However, to guarantee robustness, RQ drops a large proportion of the generated local clusters. Hence, it requires high average degree to compensate the shortage of the local clusters.

The LAS algorithm gets the lowest SDEE and SPEE, because it does not base on clusters. Clusters may introduce errors both in generation step and in coordinate system conversion step. In generation step, the local coordinate system is generated by the inter-node distances. When the inter-node distances contain errors, the corresponding local coordinate will be non-orthogonal and this can further influence all nodes in the cluster. In the coordinate system conversion step, the conversion is purely based on the position of anchors. This procedure will introduce additional errors for the nodes that are far from the anchor nodes, and the error of the conversion will be linearly amplified by the distance to anchors.

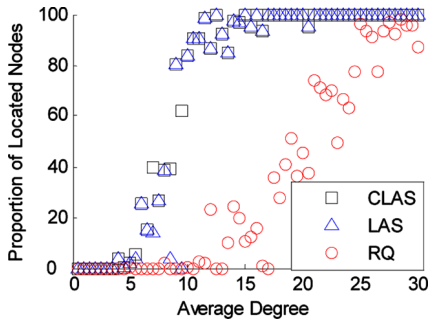
5.3 The impact of average degree

In this section, we investigate the impact of average degree. We fix the anchor proportion to 10% and set the errors as much as 10% of distance measurement. We use an empirical formula to control the average degree of the network instances into approximately linear distribution. We set the step length to be about 0.5 and report the result in Figure 6.

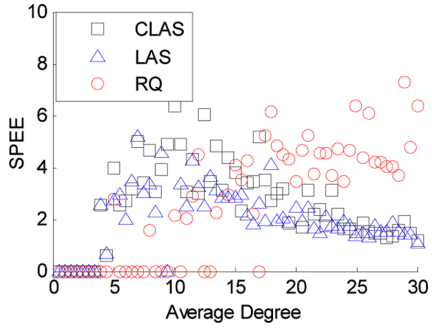
Figure 6(a) plots the proportion of localised nodes against average degree. When the average degree increases, all algorithms perform better. The LAS and CLAS algorithm can achieve 100% localisation when the average degree is greater than 12. The CLAS algorithm outperforms the LAS algorithm when the average degree varies from 5 to 10. In contrast, the RQ algorithm requires the average degree to be over 25 for full localisation. RQ requires generating uniquely overlapped local clusters, thus RQ requires higher network density than the other two algorithms.

Figure 6(b) and (c) plots the SPEE and SDEE against average degree, respectively. The SPEE and SDEE of the LAS and CLAS algorithm decrease when the average degree enlarges. Nevertheless, the SPEE and SDEE of RQ are quite stable over all the tested range. This shows that the RQ algorithm cannot benefit from the increase of network density. No matter how many neighbours are available, RQ always generates four-node clusters and uses the six inter-node distances to form local coordinates. In contrast, LAS and CLAS can perform a more accurate estimation through all available ranging measurements, which provides better estimation than RQ does.

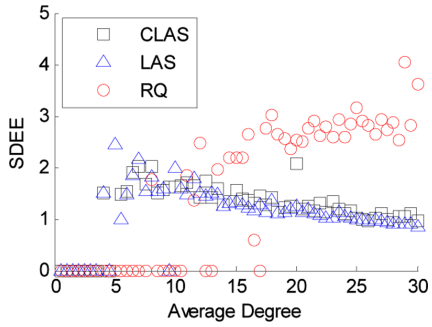
Figure 6 The impact of average degree: (a) performance; (b) position estimate error and (c) distance estimate error (see online version for colours)



(a)



(b)



(c)

5.4 The impact of anchor proportion

In this section, we investigate the impact of anchor density. We fix the average degree about 15 and set the errors as much as 10% of distance measurement. Figure 7 reports the mean value of 50 network instances in each anchor density configuration.

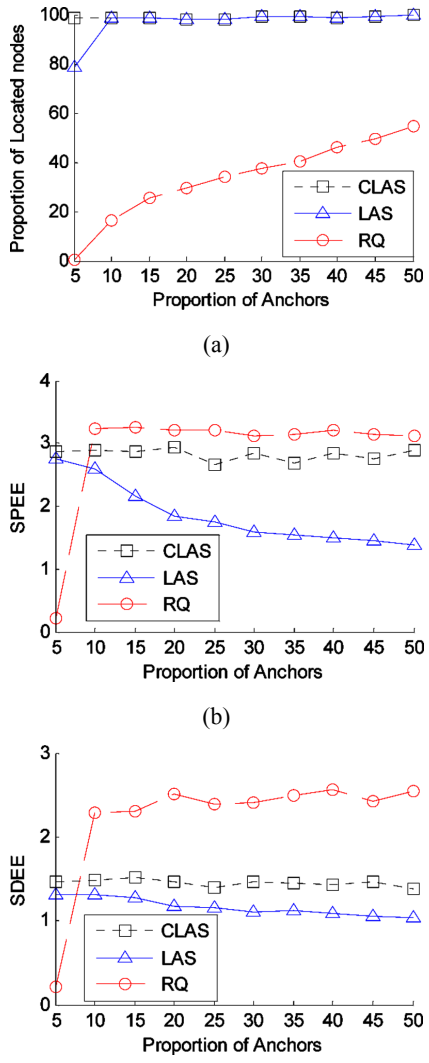
Figure 7(a) plots the proportion of located nodes against anchor density. CLAS outperforms LAS when anchors are not adequate. For a high anchor density, they both can locate almost all nodes in the network and keep this level when anchor density increases. RQ performs better when anchor density enlarges, because more anchors can help to localise the local clusters.

Figure 7(b) plots the SPEE against anchor density. The SPEE of LAS algorithm decreases when more anchors exist. However, the CLAS and RQ algorithm do not benefit from anchor density increase. The cluster-based method has the inherent limitations on the accuracy of the coordinate

system generation and localisation procedure as discussed before, and these errors cannot be reduced by anchors.

Figure 7(c) plots the SDEE against anchor density. All the algorithms have little change when anchor density varies, because SDEE indicates the mean distance estimation error. This error is dominated by the error of distance measurement, so that the anchor density does not influence it. We observe that both SPEE and SDEE of RQ are quite low at the point 5% anchors. We define the SPEE and SDEE is zero when no nodes are successfully localised, thus the values of SPEE and SDEE will be low when RQ fails locating some of network instances.

Figure 7 The impact of anchor proportion: (a) performance; (b) position estimate error and (c) distance estimate error (see online version for colours)



5.5 *The impact of error magnitude*

In this section, we investigate the impact of error magnitude. We fix the anchor proportion to 10% and set the average degree about 25. Figure 8 reports the mean value of 50 network instances in each configuration of the maximum error proportion.

Figure 8 The impact of error magnitude: (a) performance; (b) position estimate error and (c) distance estimate error (see online version for colours)

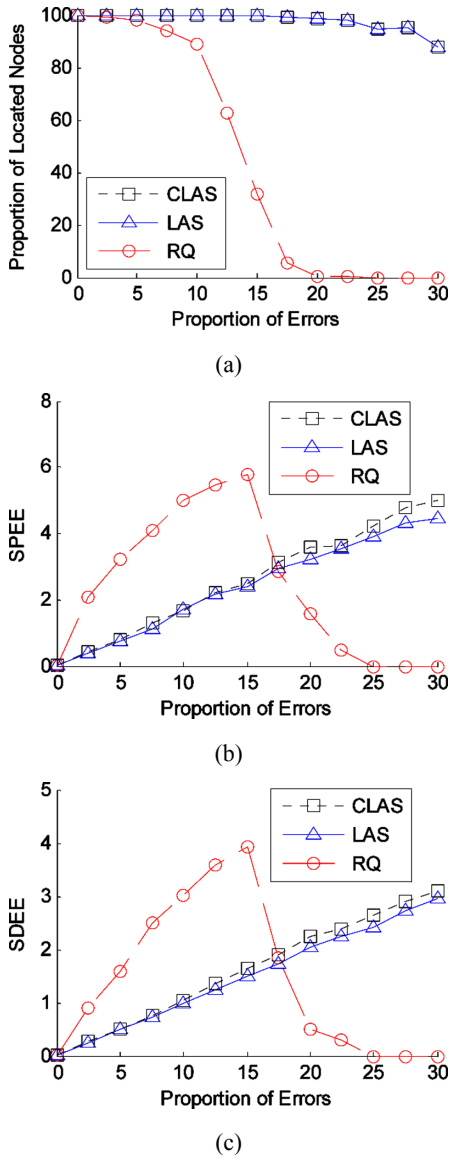


Figure 8(a) plots the proportion of located nodes against the standard deviation of the ranging error. All the algorithms perform worse when the measurement errors enlarge, because higher error will make more trilateration non-robust for flip ambiguities, thus to prevent some nodes to be located. High error affects RQ more, because RQ relies on overlapped local clusters to process localisation. Higher error will affect the generation of local clusters in cluster generation step, and then influence the whole following procedure.

Figure 8(b) and (c) plots the SPEE and SDEE against error magnitude, respectively. For the LAS and CLAS, the SPEE and SDEE are approximately linear with the error magnitude. The SPEE and SDEE of RQ are linear with the error, when the error is less than 15% of the distance measurement. Then, they start to decrease, because the SPEE and SDEE will be zero when no nodes are located in some of the network instances.

6 Related work

Localisation in wireless networks has attracted significant research interest in recent literatures (Liu et al., 2010). In this section, we briefly review some typical works in this area, including the localisation theory and the error control algorithms.

6.1 Theory of network localisation

Recently, a theory of network localisation is proposed to investigate network localisation problem (Aspnes et al., 2006). First, Aspnes et al. (2004) show the NP-hardness of the network localisation problem and propose to investigate the localisation problem in sparse networks. Then, Eren et al. adopt rigidity theory to investigate the network localisability problem (Eren et al., 2004; Goldenberg et al., 2005). Afterwards, Fang et al. propose to bridge the gap between the localisable network and location computation by the concept of sequential localisation (Fang et al., 2009). Finally, based on localisation theory, a series of algorithms are proposed to achieve efficiency on localisation (Goldenberg et al., 2006; Wang et al., 2008). In addition, some algorithms also utilise rigidity theory (Lederer et al., 2008; Wang et al., 2009; Priyantha et al., 2005) to achieve other design goals. Nevertheless, rigidity theory is not so applicable to model practical localisation problem, because the fundamental assumption of rigidity theory is that the distances between the nodes are accurate. The ranging measurement, however, always contains errors in practice (Moore et al., 2004). To the best of our knowledge, there are no works that adopt the practical model for the localisability problem. In a word, our design is an extension on both theory and algorithm of network localisation.

6.2 Error control in localisation

There is a large category of localisation algorithms aiming to diminish the errors in the result (Ni et al., 2004; Shang and Ruml, 2004; Lim and Hou, 2005; Niculescu and Nath, 2004; Savvides et al., 2003, 2005; Whitehouse et al., 2005; Li and Liu, 2010; Liu et al., 2007, 2008), when the ranging information is not accurate. Moore et al. (2004) propose the concept of robust quadrilaterals to avoid flip ambiguity with noisy ranging measurement. They choose robust geometric structures in the network as the basic localisation units to guarantee the structural uniqueness of located nodes. Besides, Kannan et al. (2007, 2008) complement this work by general geometric analysis. To the best of our knowledge, this design is the most recent work to achieve structural uniqueness with noisy inter-node distances. However, their approach is too aggressive for flip ambiguity prediction, so that it will greatly decrease the localisation performance. Moreover, simulation result shows that based on the small-scale clusters will suffer error accumulation in the result. In contrast, LAS can achieve structural uniqueness as well as high localisation performance.

Liu et al. (2006) utilise a probabilistic model to estimate and control errors of trilateration. They evaluate the possible errors in each trilateration and select robust reference nodes to minimise the estimated error of the result (Yang and Liu, 2010). Their design mainly focuses on the overall estimation error of the result. Nevertheless, the estimation of error will not exclude the structural deformation of the result, thus their approach provides no guarantee on the localisability of the result. In contrary, LAS can guarantee the structural uniqueness of the result and the error of the result is proportional to the ranging errors. Fortunately, we can adopt error control in LAS to benefit from both of the advantages, i.e., the structural stability and better error control.

Basu et al. (2006) propose an iterative method to locate nodes with noisy distance and angle information. They use linear programming to address the relaxed form of the raised problem. Their design can also guarantee to avoid structural deformation and also provide the error estimation in the result set. However, this design relies on the knowledge of both distance and angle measurement of the neighbouring nodes, which are not easy to acquire in practice.

Some researchers utilise the Cramer-Rao Lower Bound (CRLB) to characterise the error of network localisation (Niculescu and Nath, 2004; Savvides et al., 2003, 2005; Patwari et al., 2005). CRLB provides a lower bound on the variance achievable of an unbiased location estimator (Patwari et al., 2005). However, all probabilistic methods, including CRLB, cannot guarantee structural uniqueness of localisation as discussed in Section 2. Further, CRLB requires the estimator to be unbiased, which is not so realistic for the off-the-shelf location estimators.

7 Conclusions and future works

In this paper, we introduce the concept of strong localisability to achieve the structural stability for networks with noisy ranging measurements. We also propose an algorithm, called LAS, to identify and locate a kind of strongly localisable network: robust trilateration network. Compared with the existing approaches, LAS introduces lower cost on performance and controls error better. Extensive experiments are conducted to evaluate the efficiency of the proposed algorithms.

The future work leads into two directions. First, we will design low complexity algorithms for EIL test of unequal radii. Second, we will investigate how to obtain strong localisability in a general network. Currently, we are implementing this design in our ongoing projects.

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